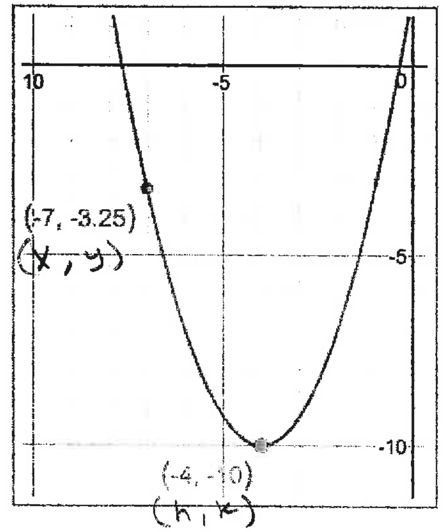
Exemplar

Unit 2 Study Guide

LT 2.1 I can create an equation that models a scenario and explain my work. (A-CED.2)

1) Write the equation of the quadratic in the graph in vertex form.



$y = a(x - -4)^2 - 10$

$y = a(x+4)^2 - 10$

$y = a(x-h)^2 + k$

$-3.25 = a(-7+4)^2 - 10$

$-3.25 = a(-3)^2 - 10$

$+10 \qquad +10$

$\frac{6.75}{9} = \frac{a(9)}{9}$

$.75 = a$

$\boxed{y = .75(x+4)^2 - 10}$

2) Write the equation of a quadratic that has a vertex of (-5,20) and contains the point (3,-2).

(h, k) (x, y)

$y = a(x - -5)^2 + 20$

$y = a(x+5)^2 + 20$

$-2 = a(3+5)^2 + 20$

$-2 = a(8)^2 + 20$

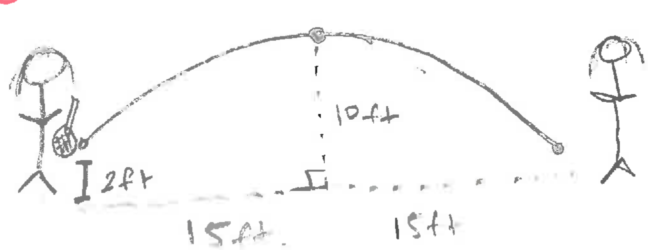
$-20 \qquad -20$

$\frac{-22}{64} = \frac{a(64)}{64} \Rightarrow a = -.34375$

$\boxed{y = -.34(x+5)^2 + 20}$

3) When a tennis ball is hit, its path through the air is almost a perfect parabola. Maria Sharapova hit a ball (2 feet off the ground) to her friend on the other side of the tennis court. The tennis ball reached its maximum height of 10 feet and at this point, the ball is right above a spot on the ground that is exactly 15 feet from where she is. Write the equation that represents the path of the tennis ball

Visual



Vertex: (15, 10)

point: (0, 2) or (30, 2)

$y = a(x-15)^2 + 10$

$2 = a(0-15)^2 + 10$

$2 = a(-15)^2 + 10$

$-10 \qquad -10$

$\frac{-8}{225} = \frac{a(225)}{225}$

$-.03\bar{5} = a$

$y = -.03\bar{5}(x-15)^2 + 10$

LT 2.2 I can make sense and explain of various parts of an equation within a context. (A-SSE.A1.b)

1) The population of a snow crab population can be represented by the equation $P(t)=2,700(1.124)^t$, where t represents the number of years after its initial recording and P(t) is the snow crab population.

A) What is the snow crab population after 3 years from its initial recording?

$P(3)=2,700(1.124)^3 \Rightarrow 3,834.09$ In 3 years there will be 3,834 snow crabs.

B) Is the population growing or decaying? Explain.

The population is growing because the growth factor is greater than 1.

C) By what percentage is the snow crab population increasing/decreasing by?

the population is increasing by 12.4% each year

$$\begin{array}{r} 1.124 \\ -\ 1.000 \\ \hline .124 \end{array}$$

12.4%

LT 2.3 I can build a function that models a scenario and solve for specific quantities. (F-BF.1)

1) a) Sergio's ant farm started with 4 ants on April 2nd. Every day the ant population is growing by 20% of its previous amount. Create an equation that describes the growth of the ant farm where A(t) represents the ant population and t is time in days.

20%
.20

$A(t)=a(1\pm r)^t$

$A(t)=4(1+.20)^t$

$\boxed{A(t)=4(1.20)^t}$

b) What was the population on April 14th?

$A(12)=4(1.20)^{12}$ 12 days later.

$A(12)=35.66$ There were 35 Ants in the population after 12 days.

c) In how many days will the population reach a total of 221 ants?

Rule
$b^x=a$
$\log_b(a)=x$

$A(t)\uparrow$

$\frac{221}{4}=\frac{4(1.20)^t}{4}$

$55.25=(1.20)^t \Rightarrow \log_{1.20}(55.25)=t$

$\boxed{22.00=t}$

In 22 days the population will reach 221 Ants

2) Mr. Santacruz is going to invest his money in a stock that will give him 1.031% every year. Determine how long it will take for Mr. Santacruz to accrue double of his initial investment of $700. ← 1400 is Double.

$f(t)=a(1\pm r)^t$

$f(t)=700(1+.01031)^t$

$f(t)=700(1.01031)^t$

1.031
.01031

$\frac{1400}{700}=\frac{700(1.01031)^t}{700}$

$2=(1.01031)^t$

$\log_{1.01031}(2)=t$

$\boxed{t=67.58}$

The account will double in 67.58 years

LT 2.4 I can explain the affect to a graph given a specific function notation. (F-BF.3)

1) Which function g(x) represents the function $f(x)=(x-3)^2+1$ after a vertical shift 2 units up and a horizontal shift 2 units to the right?

↑ ↑ -2 +2   2↑   2→

A) $g(x)=(x+1)^2+3$

B) $g(x)=(x+2)^2+3$

C) $g(x)=(x-5)^2+3$

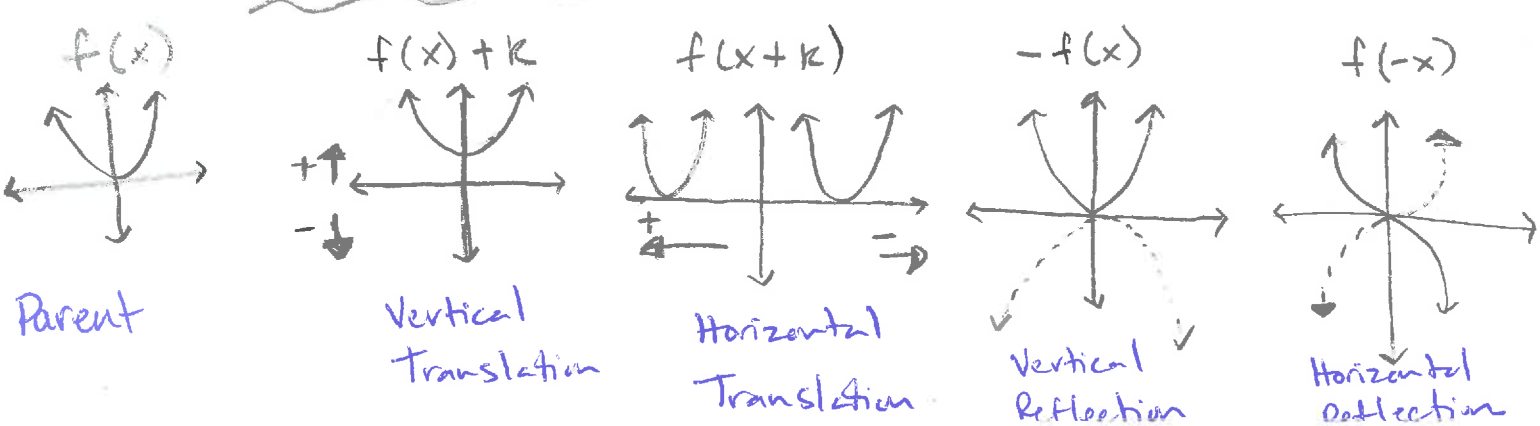
D) $g(x)=(x-1)^2+3$

2) The parent function for a quadratic is represented by $f(x)=x^2$.

For each function in the table, describe how the parent function was transformed. Be specific in the number of units and the direction of each transformations. USE ACADEMIC VOCABULARY.

Function Notation	Description
↑4 $f(-x)+4$	① Vertical translation of 4 units up. ② Horizontal Reflection across the y-axis.
←2 $-f(x+2)$	① Horizontal translation of 2 units left ② Vertical Reflection across the x-axis
→4 ↑2 $f(x-4)+2$	① Horizontal translation of 4 units Right ② Vertical translation of 2 units up.

Transformations



LT 2.5 I can solve for the key features of a graph and relate them to a context. (A-REI.4)

1) Find the roots of the following quadratic:

a) y= $6x^2 + 9x - 27 = 0$

$\boxed{3x}$ $\boxed{9}$ $18x$

$\boxed{2x}$ $\boxed{-3}$ $-9x$

$9x$ ✓

$(3x+9)(2x-3) = 0$

$3x+9=0$ $\quad 2x-3=0$

$-9 \quad -9$ $\quad +3 \quad +3$

$\frac{3x}{3} = \frac{-9}{3}$ $\quad \frac{2x}{2} = \frac{3}{2}$

$\boxed{x=-3}$ $\quad x=1.5$

$\boxed{(-3,0) \text{ \& } (1.5,0)}$

b) y= $6x^2 + 11x + 4$

$\boxed{3x}$ $\boxed{4}$ $8x$

$\boxed{2x}$ $\boxed{1}$ $3x$

$11x$ ✓

$(3x+4)(2x+1) = 0$

$x = -\frac{4}{3}$ $\quad x = -\frac{1}{2}$ $\quad (-\frac{4}{3}, 0) \text{ \& } (-\frac{1}{2}, 0)$

2) Albert tosses a ball from a high bridge. The time, t, in seconds it takes for the ball to reach the ground can be found using the equation below.

$$0 = -16t^2 + 11t + 87$$

Use the quadratic formula to find t, the time it takes for the ball to reach the ground. The answer will be positive.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=-16 \quad b=11 \quad c=87$

$x = \frac{-11 \pm \sqrt{(11)^2-4(-16)(87)}}{2(-16)} = \frac{-11 \pm \sqrt{121+5376}}{-32} = \frac{-11 \pm \sqrt{5497}}{-32} = \frac{-11 \pm 74.14}{-32}$

$+ \quad \frac{-11+74.14}{-32} = \frac{63.14}{-32} = -1.97 \quad (-1.97, 0)$

$- \quad \frac{-11-74.14}{-32} = \frac{-85.14}{-32} = 2.66 \quad (2.66, 0)$

It took the ball 2.66 seconds to reach the ground.

3) Determine which quadratic equation has 0,1, or 2 roots.

a) $f(x) = 3x^2 - 4x + 2$

b) $f(x) = -2x^2 + 6x - 8$

c) $f(x) = 9x^2 + 24x + 16$

Disriminant

$b^2-4ac > 0$ → 2 solns

$b^2-4ac = 0$ → 1 solns

$b^2-4ac < 0$ → 0 solns

a) $(-4)^2 - 4(3)(2)$

$16 - 24$

-8

No Real Solutions

b) $(6)^2 - 4(-2)(-8)$

$36 - 64$

-28

No Real Solutions

c) $(24)^2 - 4(9)(16)$

$576 - 576$

0

1 Real Solution

4) Find the Complex roots of the following quadratics ***HONORS ONLY***

a) $f(x) = x^2 + 4x + 5$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$

$\sqrt{-4} \to \sqrt{4} \; \sqrt{-1} \to 2 \; i$

$x = \frac{-4 \pm \sqrt{-4}}{2}$

$x = \frac{-4 \pm 2i}{2} \div 2$

$\boxed{x = -2 \pm i}$

b) $f(x) = x^2 - 4x + 13$

$x = \frac{+4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$

$\sqrt{-36} \to \sqrt{36} \; \sqrt{-1} \to 6 \; i$

$x = \frac{4 \pm \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2}$

$\frac{4 \pm 6i}{2} \div 2 = \boxed{2 \pm 3i}$

Vertex:

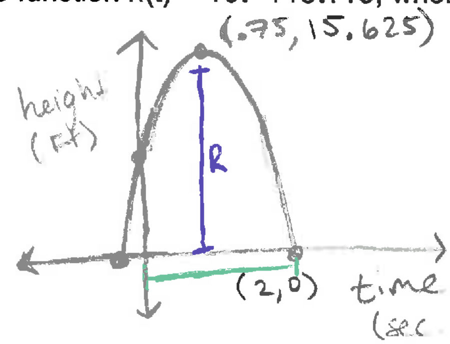
$x = \frac{-b}{2a}$ $\frac{-15}{2(-10)} = \frac{-15}{-20}$

$x = .75$

LT 2.6 I can determine the domain and range of a graph given the context. (F-IF.5)

<u>*HONORS ONLY*</u>

When a ball bounced, its height above the ground was described by the function $h(t) = -10t^2 + 15t + 10$, where t = time in seconds and h(t) = height in feet.



a) What is the domain of this function in this situation?

Set: [0, 2]

Interval: $0 \le t \le 2$

b) What is the range of this function in this situation?

Set: [0, 15.625]

Interval: $0 \le h(t) \le 15.625$

LT 2.7 I can calculate the average rate of change from many representations (Table, Graph, and Symbols) and explain what it means. (F-IF.6)

a) During a rainy spring season, Danny notices that the lawn at her house is growing at a rapid pace. She measures its height and records her findings in the table below:

Day	Height (cm)
1	5
2	6
3	8
4	13
5	15

(3, 8)
a f(a)
(5, 15)
b f(b)

AROC: $\frac{15-8}{5-3} = \frac{7}{2}$ $\frac{cm}{day}$

$\boxed{\text{AROC: } \frac{7}{2} \frac{cm}{day}}$

What is the average rate of change, in centimeters (cm) per day, of the height of the lawn from day 3 to day 5?

$\frac{f(b) - f(a)}{b - a}$

b) What is the average rate of change for the function $f(x) = x^2 + 4x$ on the interval $-6 \le x \le -2$?

a b

(−6, 8)
a f(a)

$f(-6) = (-6)^2 + 4(-6)$
$36 - 24$
$f(-6) = 8$

(−2, −4)
b f(b)

$f(-2) = (-2)^2 + 4(-2)$
$4 - 8$
$f(-2) = -4$

AROC: $\frac{-4-8}{-2+6} = \frac{-12}{4}$

$\boxed{\text{AROC: } -3}$

c) A rock is falling off a cliff. Its height above the ground changes according to the equation $h = -16t^2 + 800$, where t is time in seconds and h is height in feet.

What value of t correctly completes the statement below?

Between $t = 0$ and $t = \square$, the height of the rock decreases at an average rate of 64 feet per second.

$(0, 800)$ $\boxed{t=4}$

a f(a)

$(?, ?)$

b f(b)

$\frac{(-16b^2+800)-800}{b} = -64$

$\frac{-16b^2}{b} = -64$

$\frac{-16b}{-16} = \frac{-64}{-16}$ $\boxed{b=4}$

$h(0) = -16(0)^2 + 800$

$h(0) = 800$

$\frac{f(b)-f(a)}{b-a} = -64$

$\frac{f(b)-800}{b-0} = -64$

$\frac{f(b)-800}{b} = -64$

let t=2

$h(2) = -16(2)^2 + 800$

$h(2) = 736$

check

$\frac{736-800}{2} \stackrel{?}{=} -64$

$-32 \neq -64$

let t=4

$h(4) = 544$

check

$\frac{544-800}{4} \stackrel{?}{=} -64$

$\frac{-256}{4} \stackrel{?}{=} 64$

$-64 = -64$ ✓

LT 2.8 I can match the graph of an exponential functions with its corresponding equation. (F-IF.7e)

2) Consider the exponential functions below. Which equations represent growth, decay, or neither? Rewrite the equation in the box.

$\left(\frac{1}{3}\right)^{-x} = \left(\frac{3}{1}\right)^{x}$

$(2)^{-x} = \left(\frac{1}{2}\right)^{x}$

Rule

If growth factor is greater than 1 ⇒ growth

If growth factor is less than 1 ⇒ decay

If growth factor is equal to 1 ⇒ neither

$y = .4\,(3)^x$

$y = 15(1)^x$

$y = 5\left(\frac{1}{5}\right)^x$

$y = 2(3)^{-x}$

$y = 2\left(\frac{1}{3}\right)^x$

Growth	Decay	Neither
$y = .4(3)^x$	$y = 5\left(\frac{1}{5}\right)^x$ $y = 2\left(\frac{1}{3}\right)^x$	$y = 15(1)^x$